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Experimental observation of Rabi splitting in effective near-zero-index media in the microwave regime

Liwei Zhang,[1] Yewen Zhang,[1,*] Yaping Yang,[1] Hongqiang Li,[1] Hong Chen,[1] and Shiyao Zhu[2]

[1]*Pohl Institute of Solid State Physics, Tongji University, Shanghai 200092, China*

[2]*Department of Physics, Hong Kong Baptist University, Kowloon Tong, Hong Kong, China*

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We report the observation of a strong coupling between an artificial "atom" and localized interface mode in the microwave regime. Transmittance is experimentally measured for the effective near-zero-index paired structures containing ε-negative and μ-negative materials made of composite right- and left-handed transmission lines. When the atom is embedded in the interface, because of the strong coupling between the atom and the interface mode, a Rabi splitting of 0.12 GHz is observed, which is in good agreement with numerical simulations. Different from the usual Rabi splitting observed in conventional cavities, the splitting modes in the effective zero-index media are invariant with the scaling change of the length.

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Rabi splitting has attracted considerable interest in the area of the interaction between matter and light (electromagnetic wave) in recent years [1–9]. When a two-level system is inserted into an optical microcavity, the resonant coupling between the transition of the two levels and the cavity mode will lead to a Rabi splitting. It is well known that the coupling efficiency is proportional to the strength of the optical field of the cavity mode, which is inversely proportional to the cavity volume [2,3]. Therefore, it is desired to achieve a cavity with a highly localized field and small volume. Recently, a transmission line resonator for microwaves has been employed to access the strong-coupling regime of the coherent interaction with a superconducting two-level system [5]. Compared to traditional resonators, the transmission line resonator has the obvious advantage of an extremely small cavity volume as the field is confined along the line. However, the reduction of the size of the resonator is limited by the length of the cavity along the direction of the line due to the well-known half-wavelength limit, which might be too long for large-scale integration of many units. The half-wavelength limit is the inevitable result of standing-wave cavities. Using metamaterials in the structure of a cavity might overcome the half-wavelength limit.

Metamaterials, including double-negative metamaterials (negative permittivity ε and permeability μ) [10,11] and single negative metamaterials (one negative and one positive for the permittivity ε and permeability μ) [12–17], have attracted intensive studies in the past few years, due to their unique electromagnetic properties and potential applications [9–18]. There are two kinds of single-negative metamaterials: one is the ε-negative (ENG) media and the other is the μ-negative (MNG) media. It was shown that the tunneling mode was found in a conjugate matched pairing structure made of ENG and MNG metamaterials (called the effective zero-index medium) [16,17]. A single-mode cavity with subwavelength size has been proposed based on this effective zero-index medium (ENG-MNG pair) [14], and the optical field of the tunneling mode is exponentially increasing from the boundary to the center of the pair, contrasted to the standing-wave form of the optical field in a conventional cavity. It was indicated that the interaction of a two-level system (such as an artificial atom) with the tunneling mode in the ENG-MNG pair will lead to unusual Rabi splitting, where the two splitting modes are invariant with the scaling change and incidence angles [9]. Here, we report the experimental observation of Rabi splitting with an artificial "atom" in the ENG-MNG pair in microwave regime.

Many approaches have been proposed to implement single-negative metamaterials [12,13,15,18]. The applicable approach is the transmission line loaded with series capacitor (C) and shunt inductor (L), known as composite right- and left-handed transmission line [18]. It is intrinsically nonresonant and consequently its loss is low. For the sake of simplicity, the composite right- and left-handed transmission lines possessing ENG (MNG) in a certain range of frequencies are termed as ENG (MNG) units. In the present experiment, ENG and MNG units have been fabricated on a FR-4 substrate (with relative permittivity of 4.75, thickness of 1.6 mm). The S parameters (or transmissions) of the units are simulated by an Agilent Advanced Design system and measured by means of an Agilent 8722ES vector network analyzer. In the experiment, the ENG unit with characteristic impedance of 50 Ω is designed to have unit length of $d_{\mathrm{ENG}}=7.2$ mm and loaded lumped elements $L_{\mathrm{ENG}}=5.6$ nH and $C_{\mathrm{ENG}}=5.1$ pF, and the MNG unit to have $d_{\mathrm{MNG}}=8.4$ mm, $L_{\mathrm{MNG}}=10$ nH, and $C_{\mathrm{MNG}}=2$ pF, respectively (see Fig. 1).

Based on lossless ENG and MNG units with the parameters given above, the effective relative permittivity and permeability can be described by [18,19]

$$\varepsilon_{ENG} = 3.57 - \frac{690.9}{(2\pi f)^2}, \quad \mu_{ENG} = 1.0 - \frac{87.6}{(2\pi f)^2} \tag{1}$$

for the ENG unit and

$$\varepsilon_{MNG} = 3.57 - \frac{331.6}{(2\pi f)^2}, \quad \mu_{MNG} = 1.0 - \frac{191.9}{(2\pi f)^2} \tag{2}$$

for the MNG unit, respectively, where f is the frequency measured in GHz. They are effectively homogenous single-

*Author to whom correspondence should be addressed. yewenzhang@online.sh.cn

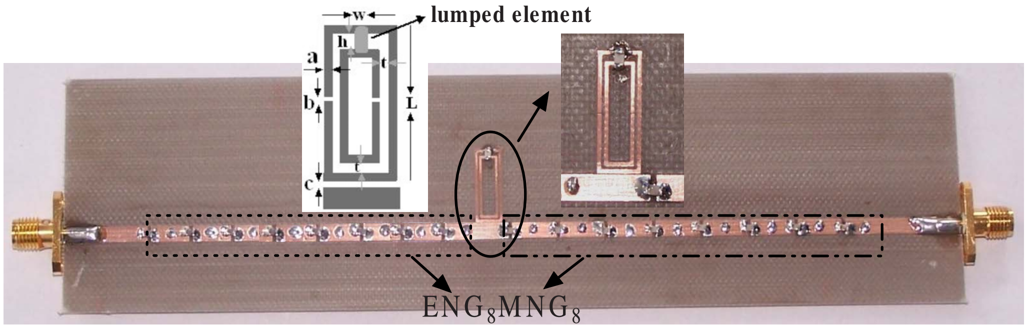

FIG. 1. (Color online) Layout of the proposed "atom"-embedded paired structure composed of ENG and MNG metamaterials based on composite right- and left-handed transmission line, where the transmission line coupled to a split ring resonator works as an artificial atom. The ENG unit parameters are unit length $d_{ENG}=7.2$ mm, loaded inductor $L_{ENG}=5.6$ nH, and capacitor $C_{ENG}=5.1$ pF; the MNG unit parameters are $d_{MNG}=8.4$ mm, $L_{MNG}=10$ nH, and $C_{MNG}=2.0$ pF; split-ring resonator parameters are $a=t=0.4$ mm, $w=5$ mm, $b=c=0.2$ mm, $L=14$ mm, and $h=0.8$ mm.

negative materials over frequencies of 1.53–2.2 GHz according to Eqs. (1) and (2). The structure made of n ENG units and n MNG units denoted by ENG_nMNG_n (see Fig. 1) possesses (near) zero average permittivity and average permeability at frequency $f_0=1.89$ GHz. A tunneling mode with zero effective wave vector emerges at normal incidence, and it localizes at the interface of the paired structure with exponentially distributed field [14,15]. For example, three orders of magnitude of enhancement of the field at the interface has been found for $n=8$ [15]. In the present experiments, the ENG and MNG units are effective single-negative materials between 1.3 and 2.1 GHz [19].

Figures 2(a) and 2(b) are the simulated and measured S parameters of the three different paired structures using real lumped elements: ENG_nMNG_n with $n=6$, 8, and 10. It is shown clearly that the three structures all possess tunneling modes at almost the same frequency f_0 where the structures are the effective zero-index medium [16]. The simulated tunneling frequency $f_0=1.74$ GHz agrees well with that of the experimental data, $f_0=1.76$ GHz. These results imply that

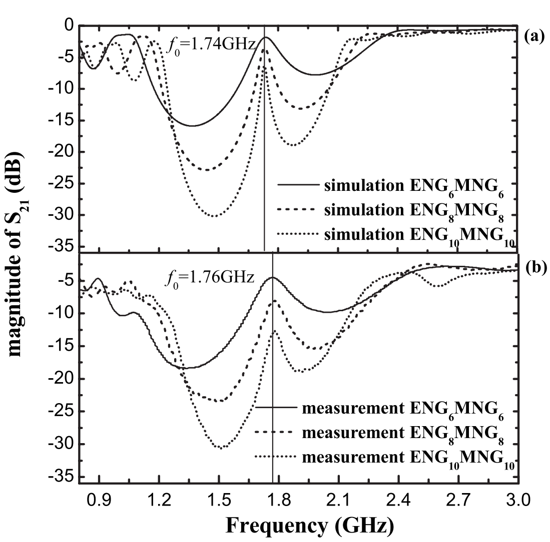

FIG. 2. The simulated (a) and measured (b) S parameters of the pairs ENG_nMNG_n with $n=6$, 8, and 10 using real lumped elements; the tunneling frequencies are 1.74 and 1.76 GHz, respectively.

the paired structures can be employed to realize a special resonator which goes beyond the half-wavelength limit since the tunneling frequency does not depend on the length of the paired structure. The quality factor increases with the structural length if we neglect the loss, signaling that the field becomes more localized, but the insertion loss increases with the length of microstrip lines and the number of real lumped elements here.

Artificial atoms based on metallic microstructures have been realized, such as "magnetic atoms" made of split ring resonators for negative permeability μ [13] and "electric atoms" made of metallic wires for negative permittivity ε [12]. For quasi-TE-wave modes guided in the transmission lines, it is experimentally convenient to use the artificial magnetic atom for probing the coupling with the tunneling mode in the paired structure. The magnetic resonance frequency f_m of the atom (the split-ring resonator fabricated on a printed circuit board) is affected by configuration parameters [20] and can be adjusted by integrating additional lumped elements [21]. When a split ring resonator is placed on one side of the transmission line, the composite structure can be also described as a composite artificial atom because of the strong magnetic coupling between the host transmission line and split-ring resonator in the low-frequency band [22]. The split-ring resonator is fabricated on FR-4 substrate shown in the inset of Fig. 3; structural parameters (see inset of Fig. 1) are $a=t=0.4$ mm, $w=5$ mm, $b=c=0.2$ mm, and $L=14$ mm, and the separation between the inner and outer rings is not uniform in order to load additional lumped elements on the upper side, where the interval is $h=0.8$ mm. There is a dip in the transmission spectrum at low frequency due to the effective magnetic resonance.

Figure 3 shows the simulated (solid circles) and measured (solid squares) f_m of the transmission line coupled to the split-ring resonator with 17 different additional surface-mount lumped elements. The left part of the horizontal axis is the loaded capacitance (0–10 pF); the right part is the loaded inductance (1.0–5.6 nH). Without additional lumped elements, the measured value of f_m is 2.22 GHz and it shifts significantly to 1.69 GHz with the loaded capacitance of 3.6 pF. But f_m decreases slightly as we continue to add the capacitance to 10 pF. If the loaded inductor is substituted for

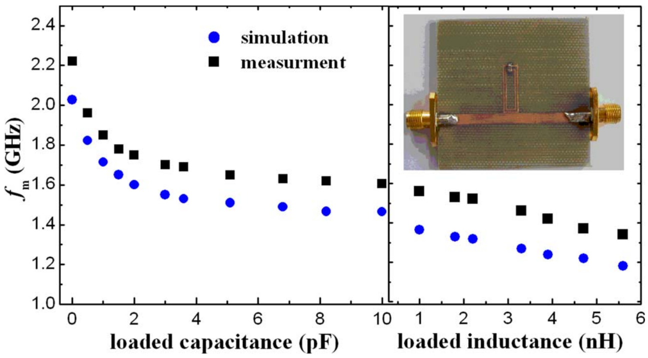

FIG. 3. (Color online) The simulated (solid circles) and measured (solid squares) magnetic resonance frequency f_m of the artificial atom with 17 different lumped elements. The left part of the horizontal axis is the loaded capacitance (0–10 pF), where 0 pF indicates no additional capacitor, the right part is the loaded inductance (1.0–5.6 nH), and the inset shows the photograph of a transmission line coupled to a split-ring resonator (working as an artificial atom).

the capacitor, f_m can also be adjusted to as low as 1.34 GHz by increasing the inductance to 5.6 nH. As seen in Fig. 3, there are some deviations between the experimental data and simulations; it is possible due to the difference between the lumped elements used in experiments and simulation and the fabricated accuracy in the artificial atom, while the simulated results also reveal the same shift trend, which agrees reasonably with the experimental data. Obviously, it is practicable to continuously tune f_m in some frequency band by loading the surface mount varactor or ceramic trimmer capacitor in the split-ring resonator [21]. If we put this kind of composite artificial atom into a cavity consisting of the paired structure with effective zero index, Rabi splitting should be observed because of the strong resonant coupling between the magnetic mode of the composite artificial atom and the tunneling of the paired structure.

Different from the standing-wave-form field in conventional cavities, the exponential-distributed field of the tunneling mode at the interface of the paired structure of ENG and MNG materials has been experimentally demonstrated [15]. When a composite artificial atom is placed in the interface of the paired structure ENG_nMNG_n, unusual Rabi splitting can be observed according to the prediction of the theoretical study of Ref. [9]. In order to study Rabi splitting and the resulting anticrossing behavior, it is necessary to tune the resonance frequency f_m of the composite artificial atom or the tunneling frequency f_0 of the ENG-MNG paired structure. Figure 4(a) presents a theoretical simulation of the transmission spectra of the artificial atom embedded in the paired structure (ENG_8MNG_8) with 14 different real lumped elements, and the data of successive simulations are offset by −10 dB. The tunneling mode in the paired structure ENG_8MNG_8 is split in the presence of the split-ring resonator: the transmission spectra of the coupled system are clearly double peaked, and the frequency splitting depends on the detuning between f_m and f_0. When decreasing f_m from 2.0 GHz to about 1.60 GHz, the upper peak f_H shifts to low frequency more quickly than that of the lower peak f_L, while

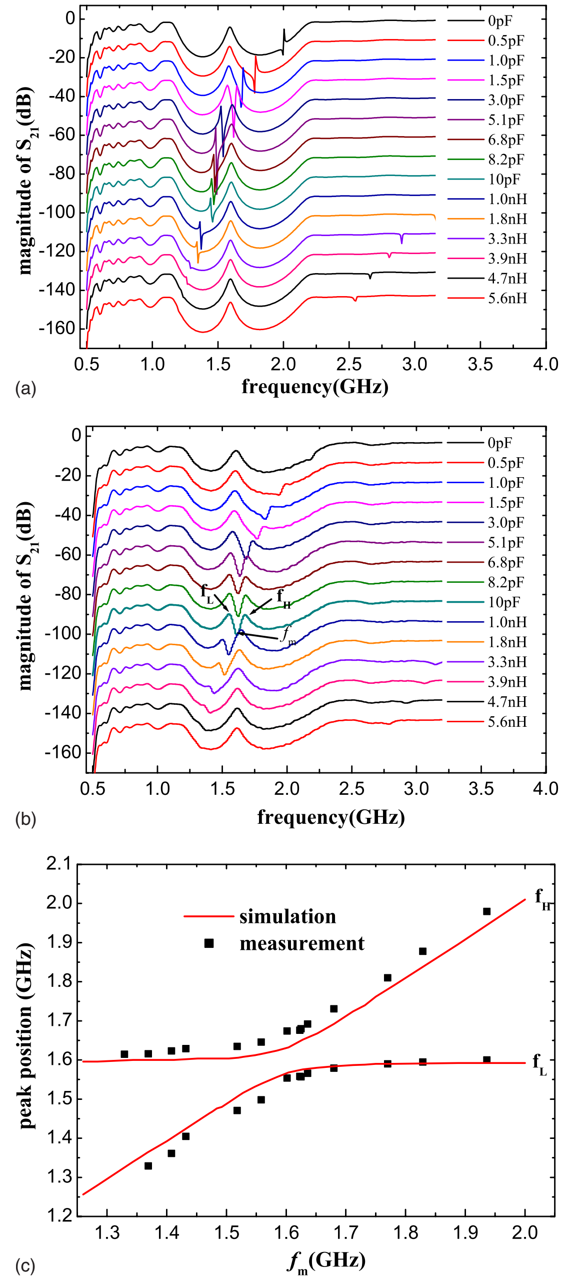

FIG. 4. (Color online) The simulated (a) and measured (b) transmission spectra of the "atom"-embedded paired structure ENG_8MNG_8 with different lumped elements; the data of successive simulations and measurements are offset by −10 dB, where f_m, f_L, and f_H represent the magnetic resonance frequency of the artificial atom and the lower and upper peak frequencies of the splitting modes, respectively. (c) Simulated (solid lines) and measured (solid squares) splitting mode peaks of the coupled system plotted as a function of f_m.

f_L shifts faster than f_H as f_m decreases further from about 1.60 GHz. The two modes repel each other in the vicinity of 1.60 GHz, showing the lumped element dependence of the resonance coupling between the split-ring resonator and the tunneling mode. Figure 4(b) shows the measured transmission spectra, revealing very similar splitting behavior as given by simulations. The splitting is always symmetric when the split-ring resonator is in resonance with the cavity mode, while the splitting and spectra shape become asymmetric at off resonance. The measured f_m of the coupled split-ring resonator in the structure is relatively higher than in the simulations when the same lumped elements are loaded, which coincides qualitatively with the uncoupled case shown in Fig. 3. The simulated and measured splitting mode peaks of the coupled system are plotted as a function of f_m in Fig. 4(c). Good agreement can be observed between the measured and simulated results. The clear anticrossing behavior is the characteristic of the Rabi splitting (the strong coupling between the tunneling mode and the magnetic mode of the artificial atom). At resonance, the experimental Rabi splitting is 0.12 GHz and the simulation result is 0.07 GHz [see from Fig. 4(c)].

The resonance condition is achieved by loading a ceramic trimmer capacitor whose capacitance can be continuously tuned with a suitable screwdriver. In order to mount the ceramic trimmer capacitor (model TZV2Z060A110, $2.3(W) \times 3.2(L) \times 1.45(H)$ mm, 2.5–6.0 pF), we adjust some parameters of the split-ring resonator shown in the inset of Fig. 1, $h=2.0$ mm and $L=15.2$ mm and keep other parameters constant. Figure 5(a) shows the simulated transmittances of the ceramic-trimmer capacitor-loaded coupled structures with $n=6$, 8, and 10 based on real lumped elements. When the double peaks are symmetrical—that is, at the resonance condition—the loaded capacitor is about 1.3 pF. It is evident that the two splitting modes are invariant with the scaling change of the structural length, while the full width at half maximum of the splitting modes becomes narrowed. Such behavior of the Rabi splitting has been predicted theoretically by a model study [9]. At the resonance condition realized by adjusting the ceramic trimmer capacitor, measured transmission spectra as the function of the structure length are given in Fig. 5(b), which are consistent with the simulations.

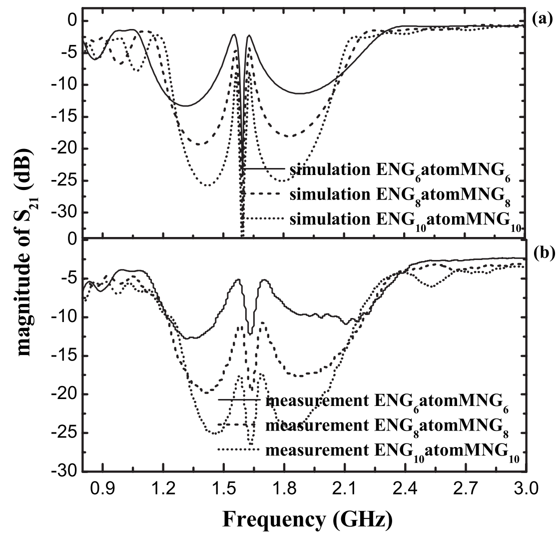

FIG. 5. The simulated (a) and measured (b) splitting modes of an artificial-atom-embedded paired structure with different ENG and MNG units at resonance condition, where the ceramic trimmer capacitor is used to change the magnetic resonance frequency of the artificial atom.

In summary, Rabi splitting of 0.12 GHz with the split ring resonator in the effective near-zero-index pair structure composed of ENG and MNG metamaterials has been observed in the microwave regime. Moreover, the splitting modes are robust against the scaling change of the matched pair structure, which is different from the usual Rabi splitting in conventional cavities.

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